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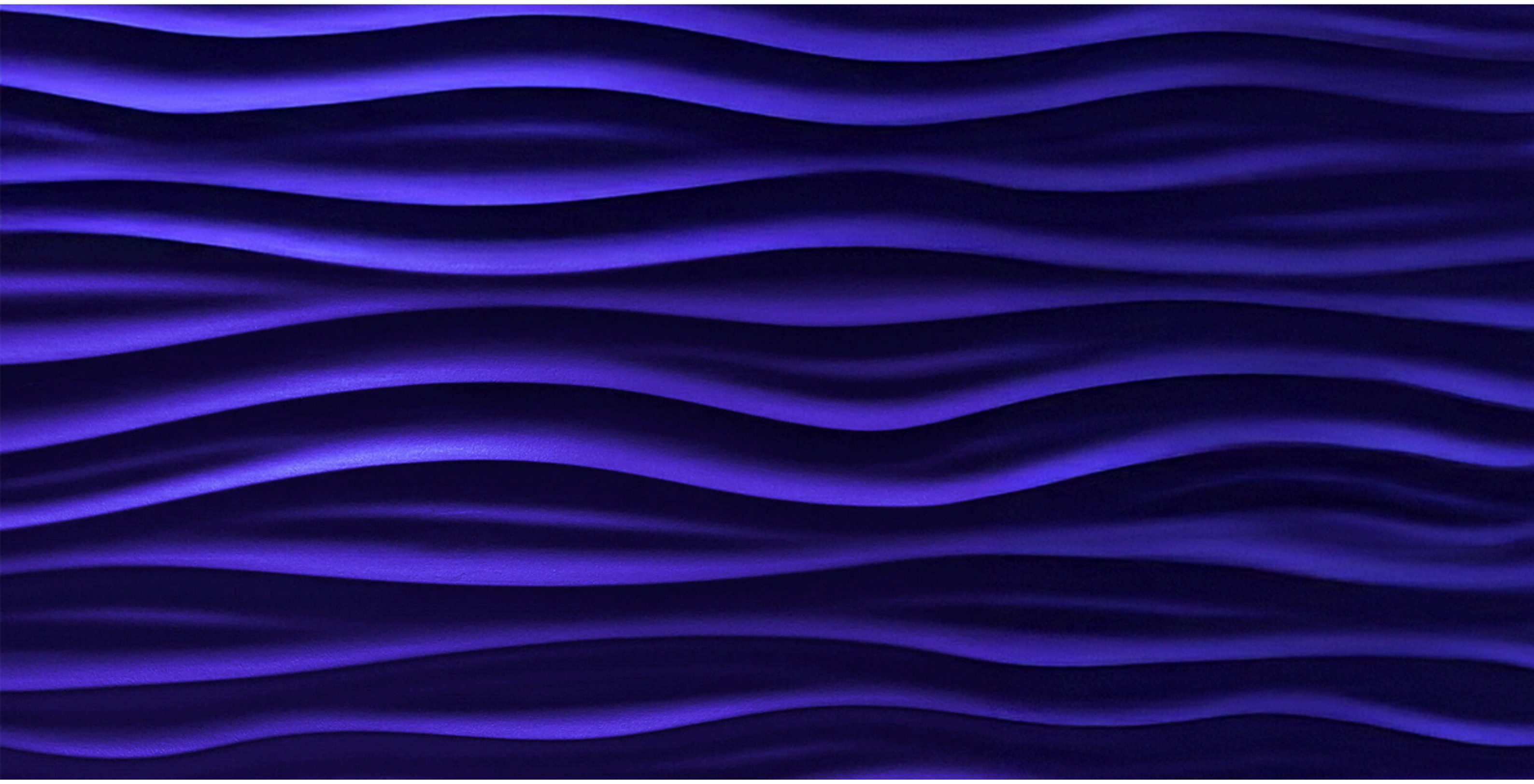

Quanscient Webinar 30th Jan 2025 | PDF summary

# Accelerating non-linear MEMS simulations with the harmonic balance method

See how harmonic balance is leveraged for solving non-linear periodic problems in the frequency domain for quicker, more precise results without transient analysis

# Contents

# Executive summary

This webinar explored the harmonic balance method, a powerful frequency-domain technique for analyzing nonlinear periodic systems.

We began with an introduction to the method, comparing it to traditional transient analysis and highlighting its advantages in terms of computational efficiency and noise reduction. We then demonstrated the practical application of the harmonic balance method within Quanscient Allsolve, our cloud-based multiphysics simulation platform, using a capacitive micromachined ultrasound transducer (CMUT) as a case study. The CMUT model setup, harmonic balance configuration, and resulting performance were presented, showcasing the spring softening effect and comparing results to transient analysis.

Finally, we extended the discussion to other real-world applications, including AC Joule heating, structural/vibrational analysis of a clamped-clamped beam, microspeakers, and loudspeakers, emphasizing the versatility and broad applicability of the harmonic balance method for complex multiphysics problems.

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# About the technical speaker

**Dr. -Ing. Abhishek Deshmukh**

Team Lead – Application Engineering

Dr. -Ing. Abhishek Deshmukh has more than a decade of experience in advanced multiphysics simulation research and software development. As the Team Lead of the Application Engineering team at Quanscient, Dr. Deshmukh works closely with customers to understand their challenges and help leverage Quanscient Allsolve to solve them.

# Introduction to Quanscient Allsolve

The cloud-based multiphysics simulation platform Quanscient Allsolve was used for all simulations featured in the webinar.

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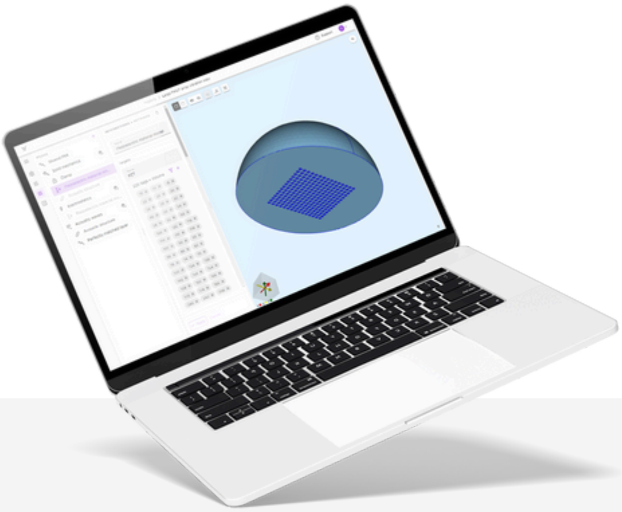

**Quanscient Allsolve**

- A cloud-based FEM multiphysics simulation platform
- Developed by Quanscient, a company established in 2021 in Tampere, Finland
- Built upon the open-source solver *Sparselizard* developed by our CTO, **Dr. Alexandre Halbach**

Trusted in both industry and academia

# Introduction to the harmonic balance method

FULL SECTION ON YOUTUBE →

## Introduction and background

The harmonic balance method is a frequency-domain technique for analyzing systems with nonlinear behavior under periodic excitation.

While linear problems are readily solved in the frequency domain, nonlinearities complicate traditional frequency analysis.

The harmonic balance method addresses this by representing the system's response as a sum of harmonic components, capturing nonlinear effects.

This is crucial for understanding devices like the electrostatically actuated spring-mass-damper system used as an example here.

Though established in the 1970s [1], the method's applicability has recently expanded due to increased computational power, particularly cloud computing platforms like Quanscient Allsolve. This enables efficient simulation of complex nonlinear systems.

## Electrostatically actuated spring-mass-damper system

Nonlinear terms:

- Mechanical motion
- Electrostatic force ∝ $V^2$

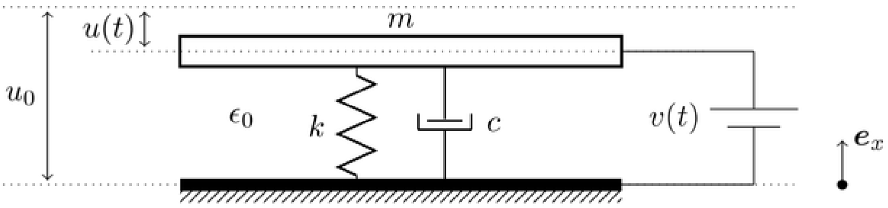

*Fig. 1*

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[1] Nakhla M. S. and Vlach J.: "A piecewise harmonic balance technique for determination of the periodic response of nonlinear systems", IEEE Transactions on Circuits and Systems, vol. 23, pp. 85-91, 1976.

# Introduction to the harmonic balance method

## Comparison to transient analysis

Transient analysis is an alternative for nonlinear systems, but has limitations:

- **Steady-state requirement:** Transient analysis requires a steady-state periodic response before extracting frequency information. This can be computationally expensive, especially with long transient periods (thousands of cycles). The vibrating disk example illustrates this (see fig. 2).
- **Manual data extraction:** Determining steady-state arrival often requires manual steps.
- **Noise:** Transient data is susceptible to noise, affecting frequency accuracy.

The harmonic balance method directly solves for the steady-state periodic response in the frequency domain, avoiding long transient simulations, manual processing, and providing cleaner results.

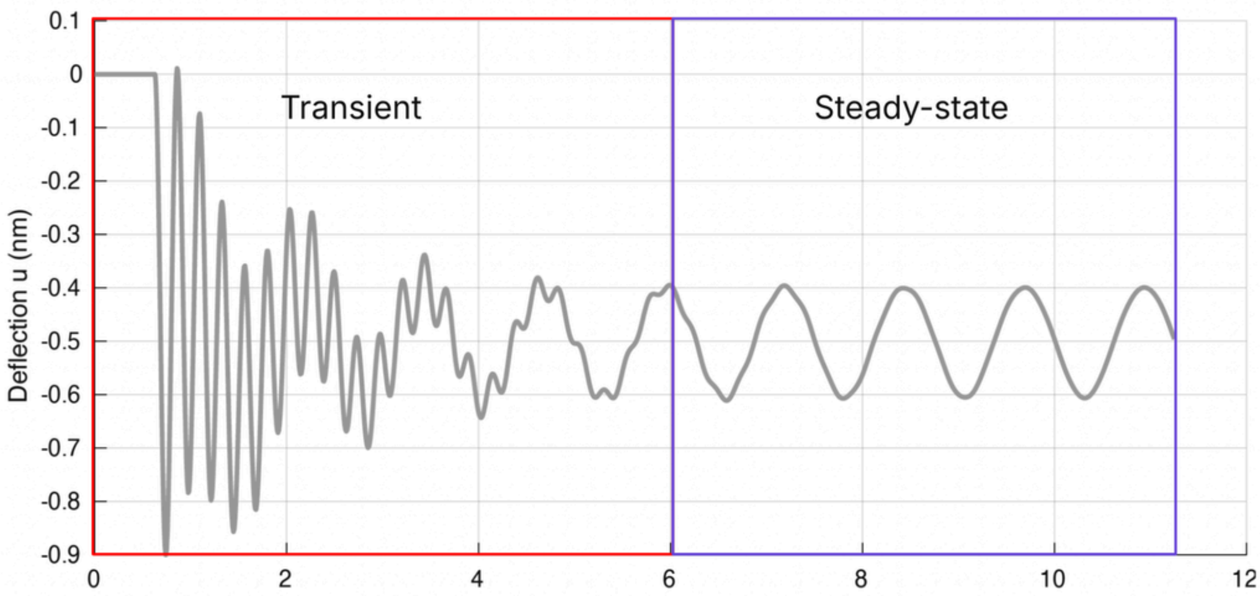


*Fig. 2*

[2] Halbach A.: "Domain decomposition techniques for the nonlinear, steady-state, finite element simulation of MEMS ultrasonic", PhD Thesis, University of Liège, 2017.

# Introduction to the harmonic balance method

## Key working principle

The harmonic balance method decomposes the system's response into a Fourier series, representing the field variable (e.g., displacement, voltage) as a sum of sinusoidal and cosine components at various frequencies:

- The field (a function of space and time) is expressed using Fourier coefficients.
- Each coefficient corresponds to a harmonic frequency.
- The number of harmonics depends on the system's nonlinearity.

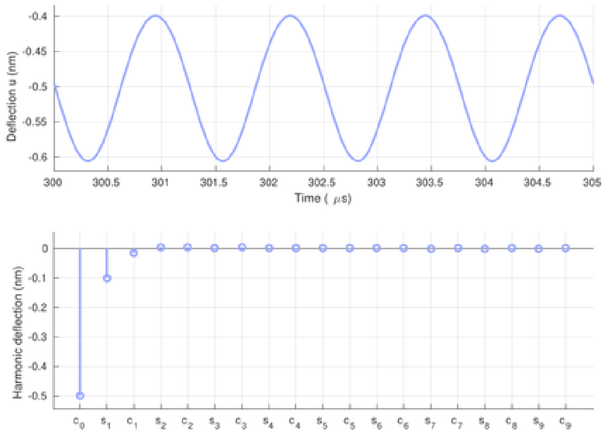
Weakly nonlinear systems may only need a few harmonics. Strongly nonlinear systems require more to capture the full response.

The spring-mass-damper example demonstrates this: a small AC voltage on a DC bias yields a nearly sinusoidal response with a dominant fundamental frequency (see fig. 3).

Increasing the AC voltage distorts the response, requiring higher harmonics (e.g., the second harmonic) for accurate representation (see fig. 4).

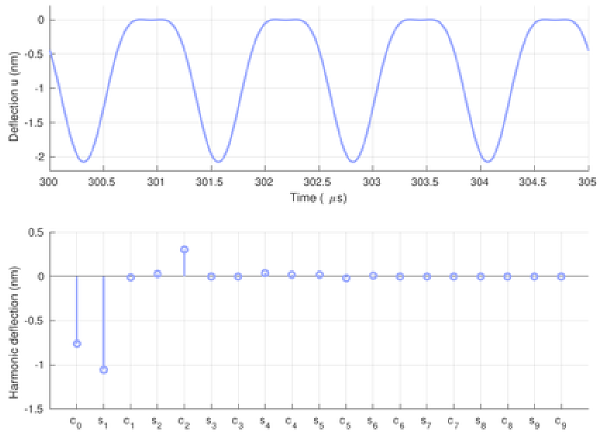
Analyzing these coefficients reveals the system's nonlinear behavior. The method balances harmonic contributions for a steady-state frequency-domain solution.

$$\phi(\boldsymbol{x},t) = \sum_{k=0}^{N} \phi_{sk}(\boldsymbol{x})\ \sin(\omega_k t) + \phi_{ck}(\boldsymbol{x})\ \cos(\omega_k t)$$



10 V DC + 1 V AC drive

*Fig. 3*



10 V DC + 10 V AC drive

*Fig. 4*

## Live demo

# Applying harmonic balance in Quanscient Allsolve

This section details the application of the harmonic balance method within Quanscient Allsolve, using a capacitive micromachined ultrasound transducer (CMUT) as an example, and compares the results to those obtained through transient analysis.

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### CMUT model setup in Quanscient Allsolve

A single-cell CMUT model was created in Quanscient Allsolve (see fig. 5) to demonstrate the harmonic balance method. The model includes:

- **Geometry**: Two parallel discs (top and bottom plates) with a vacuum cavity between them. Key dimensions like plate thickness and radius were parameterized.
- **Materials**: Monocrystalline silicon for the top plate, silicon nitride for the bottom, and vacuum for the gap.
- **Physics**: Solid mechanics (for the plates), electrostatics (for the gap), and mesh deformation (to account for plate movement).
- **Boundary conditions**: The bottom plate was clamped, the top plate's movement was constrained (only in the z-direction), and a spring was attached to the top plate via a lumped port. Electrostatic force coupling, damping, and large displacement effects were also included. A sinusoidal AC voltage was applied on top of a DC bias voltage to the cavity.

### Harmonic balance setup

After meshing the geometry using an extruded mesh suitable for thin MEMS structures (see fig. 6), a frequency sweep simulation was added. The "multiharmonic" solver (Quanscient Allsolve's implementation of the harmonic balance method) was selected.

- **Harmonics**: Constant, sine, and cosine harmonics were chosen for both the fundamental frequency and twice the fundamental frequency.
- **Frequency sweep**: A linear frequency sweep was defined, and a DC bias voltage was specified.
- **Outputs**: Harmonic magnitudes were defined as outputs for evaluation, as well as the normalized frequency.

A double sweep was then set up to analyze the CMUT's behavior across a range of frequencies and DC bias voltages, leveraging Quanscient Allsolve’s *repmat* and *repelement* functionality to efficiently create a matrix of simulations.

# Live demo

# Applying harmonic balance in Quanscient Allsolve

## Results and comparison with transient analysis

The simulations were submitted to the cloud and completed rapidly.  Results showed the expected shift in resonant frequency with increasing bias voltage, demonstrating the spring softening effect (see fig. 7).  This behavior is characteristic of CMUTs and arises from the system's nonlinearities.

Comparing the harmonic balance results to transient analysis highlighted the method's efficiency. While the harmonic balance simulations took seconds to minutes, transient simulations for similar scenarios required several hours per simulation.

Furthermore, the harmonic balance method directly provided the steady-state response in the frequency domain, whereas transient analysis required post-processing to extract this information.

For the CMUT example, transient analysis required up to 2000 cycles to accurately capture the system's behavior (see fig. 6), making the harmonic balance method significantly faster and more practical for this type of analysis.

The clear benefit of using the harmonic balance method for analyzing nonlinear periodic systems like CMUTs was thus demonstrated

### Single CMUT cell

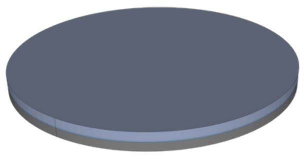

*Fig. 5*

### Transient vs. Harmonic balance

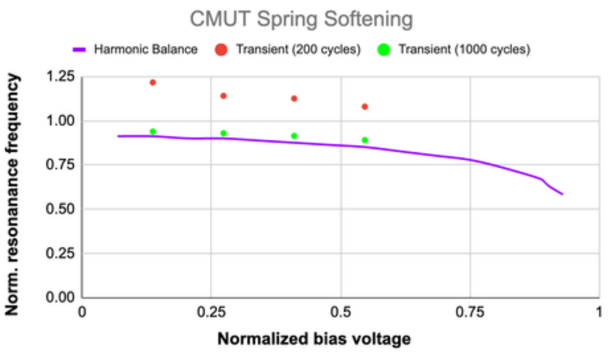

*Fig. 6  One transient simulation for 1000 cycles takes about 3 hours.*

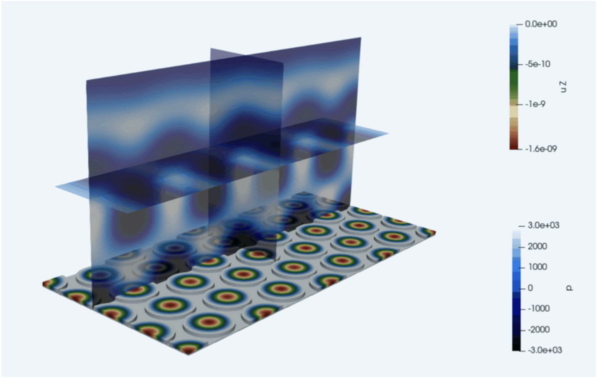

*Fig. 7*

# Other applications and real-world results

The harmonic balance method, beyond its application to CMUTs, is a versatile technique applicable to a range of nonlinear periodic problems. Here are some examples:

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## AC Joule heating

AC Joule heating, commonly seen in household appliances and electronics, involves heat generation due to alternating current flowing through a resistive conductor.

Transient simulations of a heated filament at 100Hz and 400Hz (see fig. 9) show the temperature stabilizing over time.

However, harmonic balance analysis provides deeper insights. A frequency sweep (1Hz to 500Hz) reveals the harmonic coefficients of the temperature (see fig. 10).

While the constant part (T1) represents the average heating, the second harmonic components (T4 and T5), present at twice the fundamental frequency, become significant and vary with frequency.

This behavior, not readily apparent in transient simulations, is due to the heat source being proportional to the square of the current, resulting in a temperature response with a constant component and a sinusoidal component at twice the fundamental frequency.

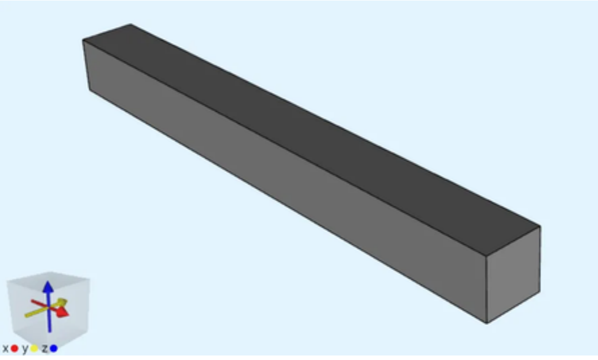


*Fig. 8*

### Transient

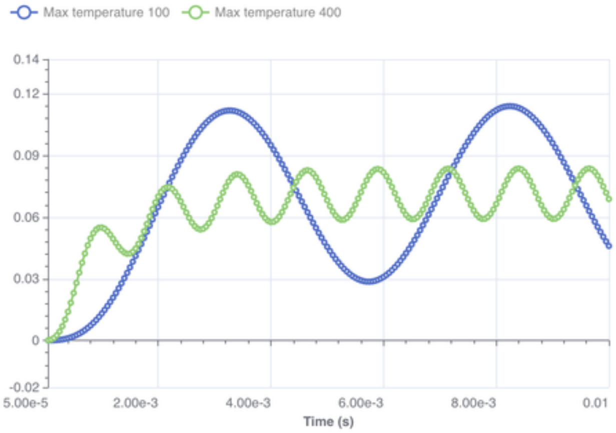


*Fig. 9 Fundamental driving frequency f0: 100 Hz, 400 Hz*

### Harmonic balance

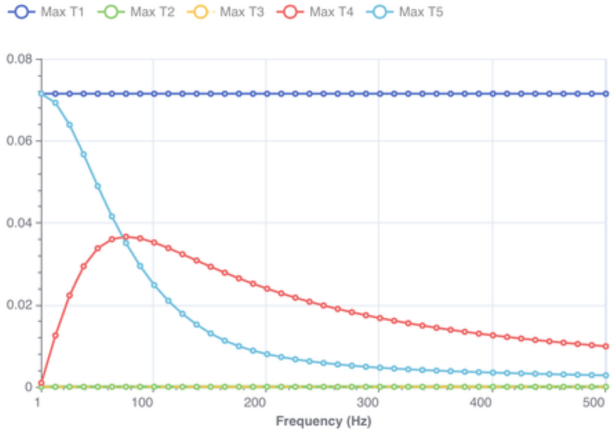


*Fig. 10 Frequency sweep: 1 Hz to 500 Hz in 40 steps*

# Harmonic balance method
# Other applications and real-world results

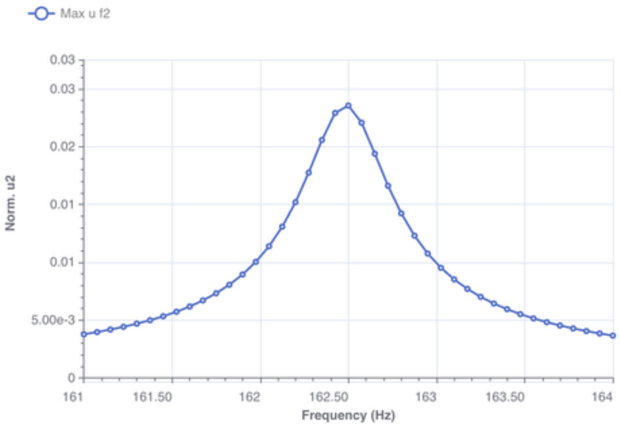
## Backbone curve: clamped-clamped beam

A clamped-clamped beam example illustrates the method's utility in structural analysis.

Linear analysis predicts a standard resonance peak (see fig. 11). However, considering geometric nonlinearity reveals multiple valid solutions at the same frequency (see fig. 12) [3] , a phenomenon difficult to capture with transient analysis.

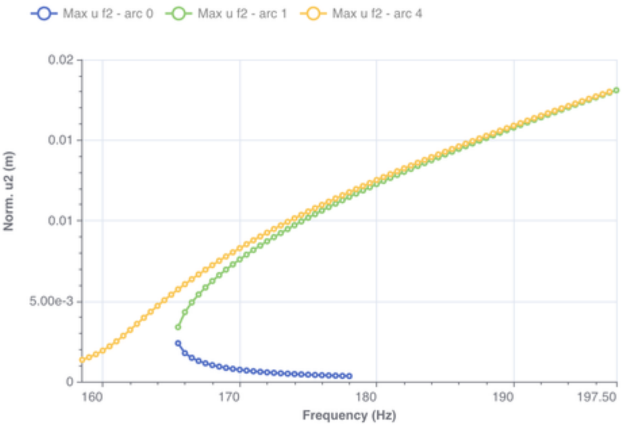
The harmonic balance method can efficiently generate this nonlinear response curve in a single simulation.

For more details on this example, please refer to our previous Harmonic Balance webinar ⟶



**Linear**

*Fig. 11*



**Nonlinear**

*Fig. 12*

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[3] Hayashi, S., Gutschmidt, S., Murray, R. et al. Experimental bifurcation analysis of a clamped beam with designed mechanical nonlinearity. Nonlinear Dyn 112, 15701–15717 (2024). https://doi.org/10.1007/s11071-024-09873-5

# Harmonic balance method
# Other applications and real-world results

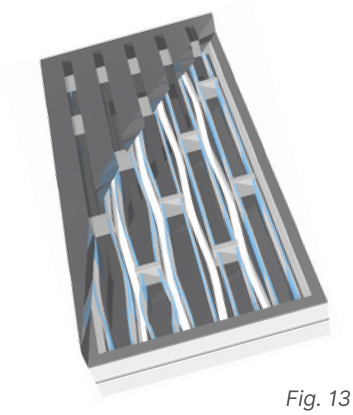


Fig. 13

## Microspeakers

Microspeakers, increasingly used for their crisp sound and control, are often electrostatically actuated.

These devices, with their moving and fixed plates, generate sound through the motion of air. Simulating their behavior requires considering fluid dynamics within small gaps, making them inherently nonlinear.

Applying the harmonic balance method to a microspeaker model (see fig. 13) allows capturing the displacement accurately, matching literature values (see fig. 14).

The method's computational efficiency enables parametric sweeps over frequency and voltage, crucial for optimizing design and performance (see fig. 15).

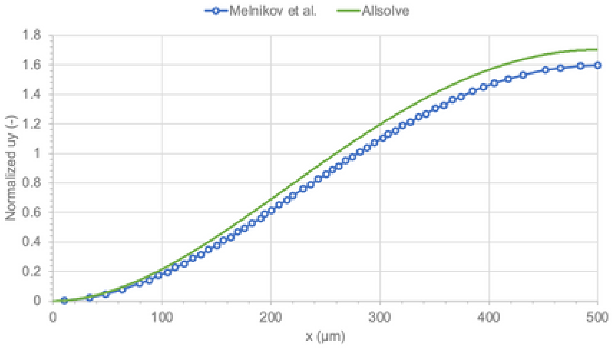


Fig. 14

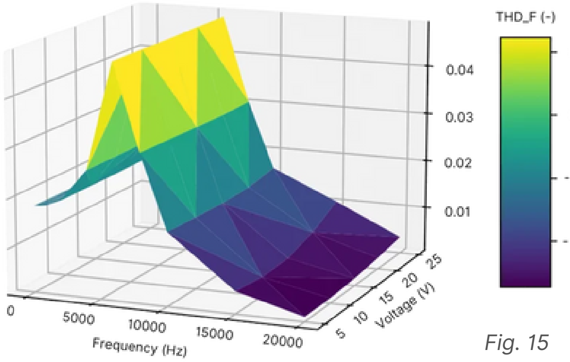
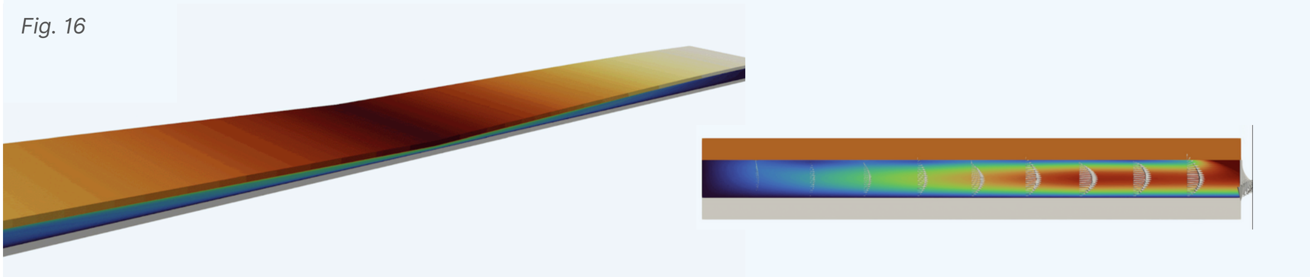


Fig. 15

### Electrostatics + Solid mechanics + Fluid dynamics

| | | | |
|---|---|---|---|
| **100** | **800k** | **<7** | **12** |
| Hz | DoFs | Minutes | Cores |

Fig. 16


[4] Kaiser, B. et al. Concept and proof for an all-silicon MEMS micro speaker utilizing air chambers. Microsyst Nanoeng 5, 43 (2019). https://doi.org/10.1038/s41378-019-0095-9

[5] Melnikov, A., Schenk, H.A.G., Monsalve, J.M. et al. Coulomb-actuated microbeams revisited: experimental and numerical modal decomposition of the saddle-node bifurcation. Microsyst Nanoeng 7, 41 (2021). https://doi.org/10.1038/s41378-021-00265-y

# Harmonic balance method

# Other applications and real-world results

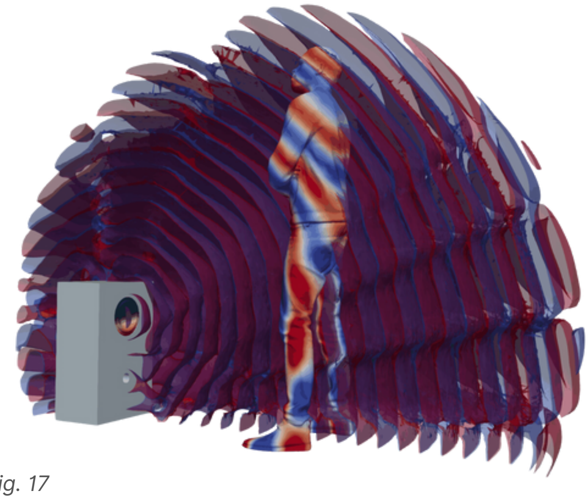


*Fig. 17*

## Loudspeakers

Finally, a loudspeaker example demonstrates the method's applicability to larger-scale systems. This complex multiphysics problem involves electromagnetics and acoustic structure interaction.

Quanscient Allsolve can handle this fully coupled analysis in the frequency domain, accounting for system nonlinearities.

When two slightly different frequencies are input to the loudspeaker (intermodulation), the output pressure signal exhibits many higher frequencies (see fig. 20). The harmonic balance method, by including these higher harmonics, readily captures this complex intermodulation effect.

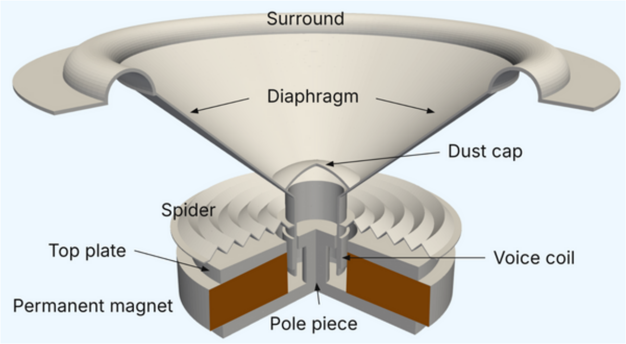


*Fig. 18*

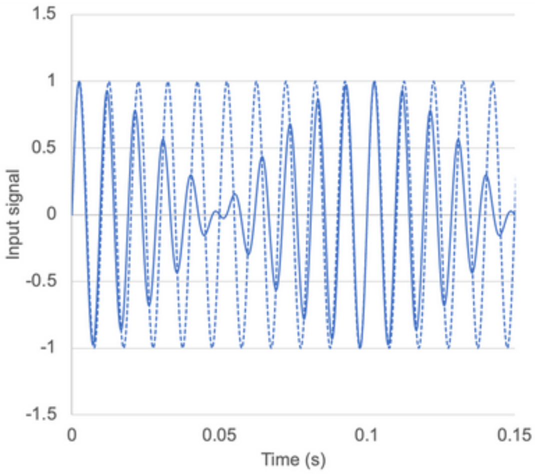


*Fig. 19*

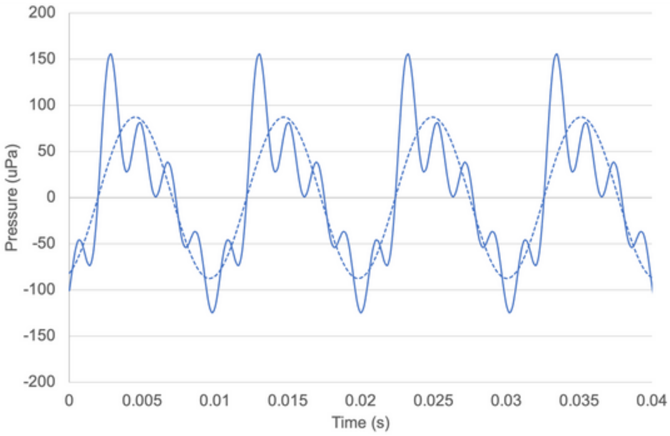


*Fig. 20*

# Conclusion and key takeaways

The harmonic balance method offers a compelling approach to analyzing nonlinear periodic systems, providing significant advantages over traditional transient analysis.
Key takeaways from this webinar include:

- **Efficiency**: The harmonic balance method directly calculates the steady-state periodic response in the frequency domain, drastically reducing computation time compared to transient analysis, which requires long simulations to reach steady state. This was clearly demonstrated in the CMUT example, where harmonic balance simulations took minutes to minutes compared to hours for transient simulations.

- **Accuracy**: By directly solving in the frequency domain, the harmonic balance method avoids the noise issues inherent in transient analysis data, leading to cleaner and more accurate results.

- **Versatility**: The method is not limited to specific applications. As demonstrated, it can be applied to a wide range of problems, from MEMS devices like CMUTs and microspeakers to larger systems like loudspeakers, and across various physics domains, including solid mechanics, electrostatics, fluid dynamics, and electromagnetics.

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- **Quanscient Allsolve integration**: Quanscient Allsolve provides a powerful and efficient platform for implementing the harmonic balance method, leveraging cloud computing resources to handle complex simulations with ease. Features like repmat and repelement further enhance efficiency for parametric studies.

- **Nonlinear insights**: The harmonic balance method allows for the identification and analysis of higher-order harmonics, providing valuable insights into nonlinear system behavior that are often missed by transient analysis. The AC Joule heating example illustrated this, revealing the presence of significant second harmonic temperature components.

**The harmonic balance method is a valuable tool for any engineer or researcher dealing with nonlinear periodic systems.**

**Its efficiency, accuracy, and versatility, especially when implemented on a powerful cloud platform like Quanscient Allsolve, make it an essential technique for analyzing and designing a wide range of devices and systems.**

# Next steps

If you are interested in exploring the capabilities of Quanscient Allsolve, we encourage you to schedule a consultation with our CRO, Mr. Nikola Strah.

**Schedule a 30-minute introductory call now.**

Book your session now!

During this meeting, you will:

- Learn how Quanscient Allsolve can address your specific simulation needs
- Learn how to integrate Quanscient Allsolve with your existing workflows
- Discover the platform's problem-solving capabilities and explore new possibilities for your use cases

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